



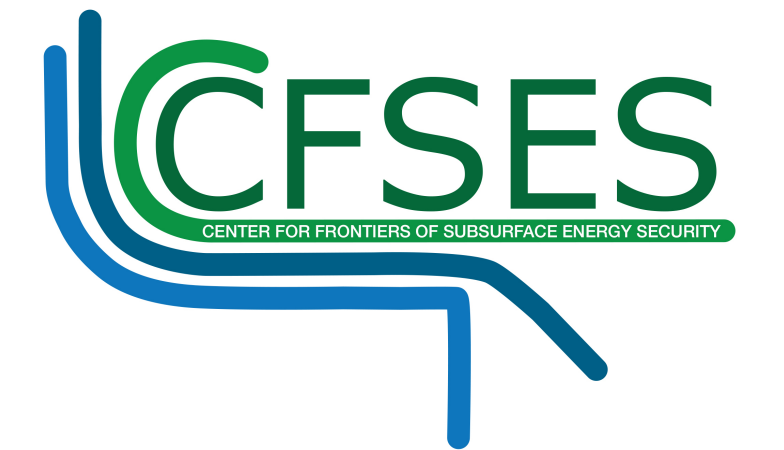
A Modeling Study of CO₂ Injection at Cranfield with Compositional Flow, Distorted Hexahedra, Elasto-Plasticity, and Stress-Dependent Permeability in a High-Performance Computing Environment

D. White[‡], B. Ganis[‡], R. Liu[‡], M.F. Wheeler[‡], T. Dewers[‡]

[‡] Center for Subsurface Modeling, Institute for Computational Engineering and Sciences, The University of Texas at Austin, Texas, USA

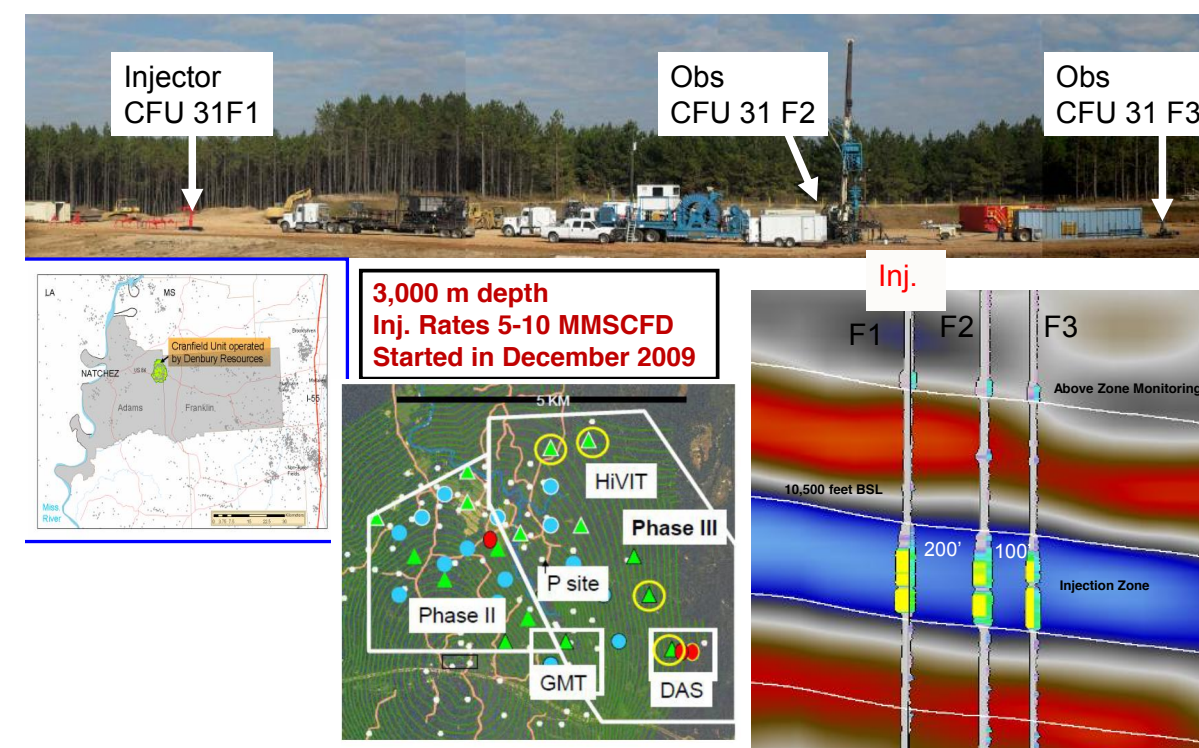
[†] Geomechanics Laboratory, Sandia National Laboratories, Albuquerque, New Mexico, USA

mfw@ices.utexas.edu



1. Introduction

- Recent history matching studies have predicted the development of high permeability channels at Cranfield when the CO₂ injection rate was doubled. [1] This suggests the need for geomechanical modeling.
- Linear elasticity is the predominant solid material model used in simulations, but nonlinear constitutive models can take into account more complex rock formation behaviors.
- Plastic behavior can occur near wellbores, resulting in changes to rock porosity and permeability, which can impact flow behavior.
- The Drucker-Prager plasticity model has been incorporated into IPARS (Integrated Parallel Accurate Reservoir Simulators developed at the Center for Subsurface Modeling, The University of Texas at Austin).
- Our models use general hexahedral elements for flow and mechanics and can solve large-scale problems in parallel.



Schematic of the Cranfield CO₂ sequestration project in western Mississippi, with wells monitored by the Bureau of Economic Geology.

2. Plasticity Model

Fluid Flow and Stress Equilibrium Equations (single-phase shown for simplicity)

$$\frac{\partial(\rho(\phi_0 + \alpha\varepsilon_v + \frac{1}{M}(p - p_0)))}{\partial t} + \nabla \cdot \left(\rho \frac{K}{\mu} (\nabla p - \rho g \nabla h) \right) - q = 0$$

$$\nabla \cdot (\sigma'' + \sigma_o - \alpha(p - p_0)I) + f = 0$$

Hooke's Law and Strain-Displacement Relation

$$\sigma'' = D^e : (\varepsilon - \varepsilon^p)$$

$$\varepsilon = \frac{1}{2}(\nabla u + \nabla^T u)$$

Yield and Flow Functions (Drucker-Prager)

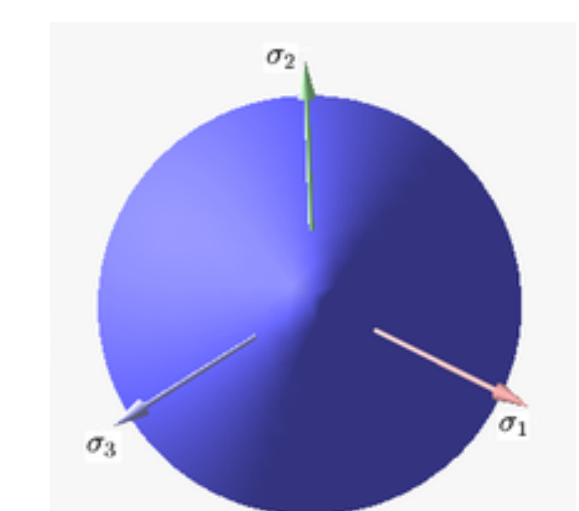
$$Y = q + \theta \sigma_m - \tau_0$$

$$F = q + \gamma \sigma_m - \tau_0$$

Plastic Strain Evolution Equations

$$\dot{\varepsilon}^p = \lambda \frac{\partial F(\sigma'')}{\partial \sigma''}, \quad \text{at } Y(\sigma'') = 0$$

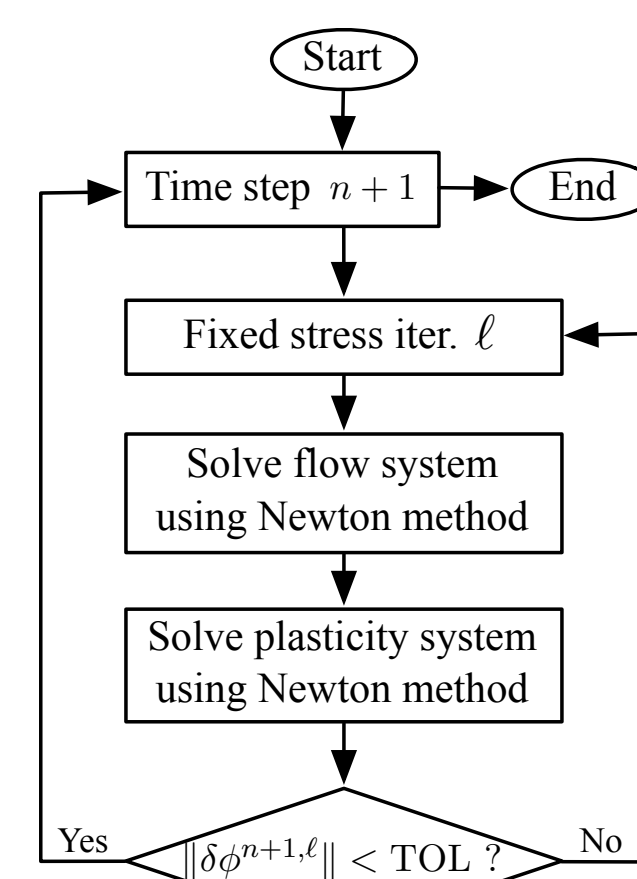
$$\dot{\varepsilon}^p = 0, \quad \text{at } Y(\sigma'') < 0$$



Drucker-Prager Yield Surface.

Here ρ is fluid density, ϕ_0 is initial porosity, α is the Biot coefficient, ε_v is volumetric strain, M is the Biot modulus, p is fluid pressure, K is permeability, μ is fluid density, $g \nabla h$ is gravitational force, q are fluid sources/sinks, σ'' is effective stress, σ_0 is initial stress, f is solid body force, D^e is the Gassman tensor, ε is elastic strain, ε^p is plastic strain, u is displacement, λ is a consistency parameter, F is plastic flow function, Y is plastic yield function, q is the Von-Mises stress, θ and γ are the yield and flow function slopes, and τ_0 is the shear strength.

- Plastic model is non-linear. A Newton iteration is used to solve the mechanics residual equations on a global level, and a second Newton iteration is used to evaluate material integration on the element level. This leads to a consistent formulation, and our numerical results show quadratic Newton convergence.
- To solve an elastic model, we may set plastic strain $\varepsilon^p = 0$, and the mechanics equation becomes linear.
- The coupled poro-plasticity system is solved using an iterative coupling scheme: the nonlinear flow and mechanics systems are solved sequentially using the fixed-stress splitting, and iterates until convergence is obtained in the fluid fraction. To the best of our knowledge, the application of this algorithm is new for plasticity.
- On any given Newton iteration, both flow and mechanics linear systems are solved using the iterative multigrid solver library HYPRE. This efficiently obtains the solution with excellent parallel scalability.



3. Numerical Results

- Our latest numerical experiments use:
 - Accurate hexahedral geometry
 - Fully compositional multiphase flow
 - Drucker-Prager poro-elasto-plasticity
 - Stress-dependent permeability
 - Rock properties from Sandia tests [2]
 - 14 injection/production wells
- High-Performance Computing Setup:
 - Jobs run on Stampede supercomputer at Texas Advanced Computing Center (TACC).
 - Parallel simulations used 512 cores across 32 compute nodes.
 - The longest runtime was 34 hours.



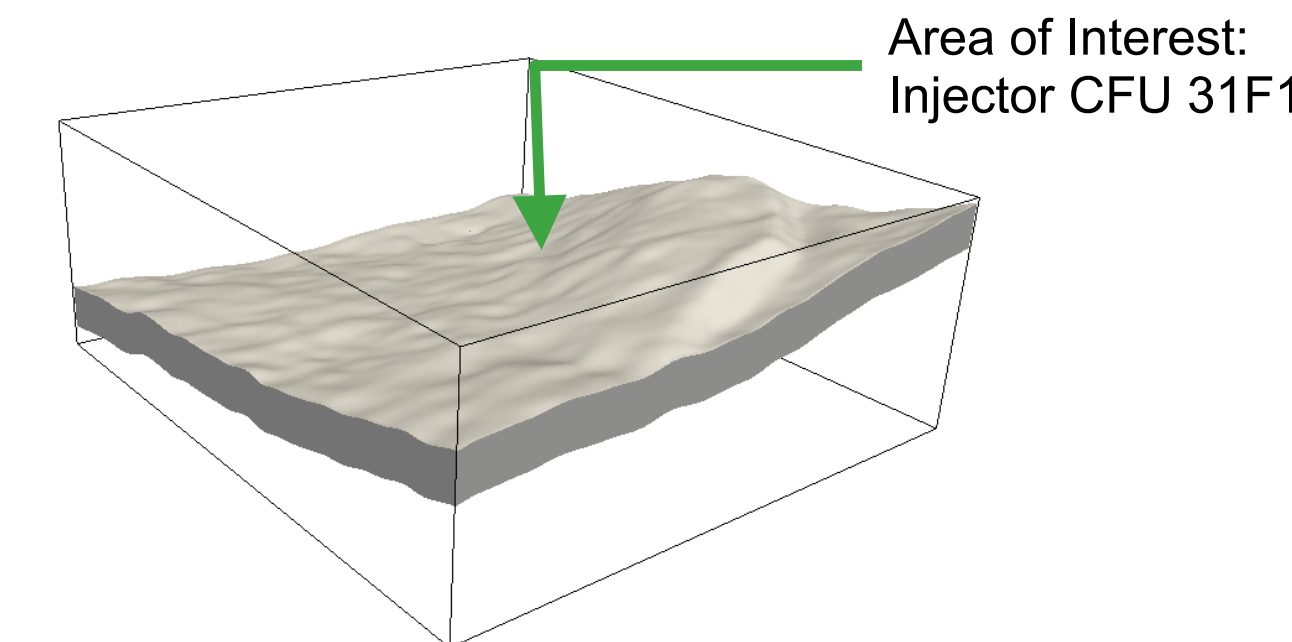
3.1 Problem Setup

- Five numerical simulations were run:

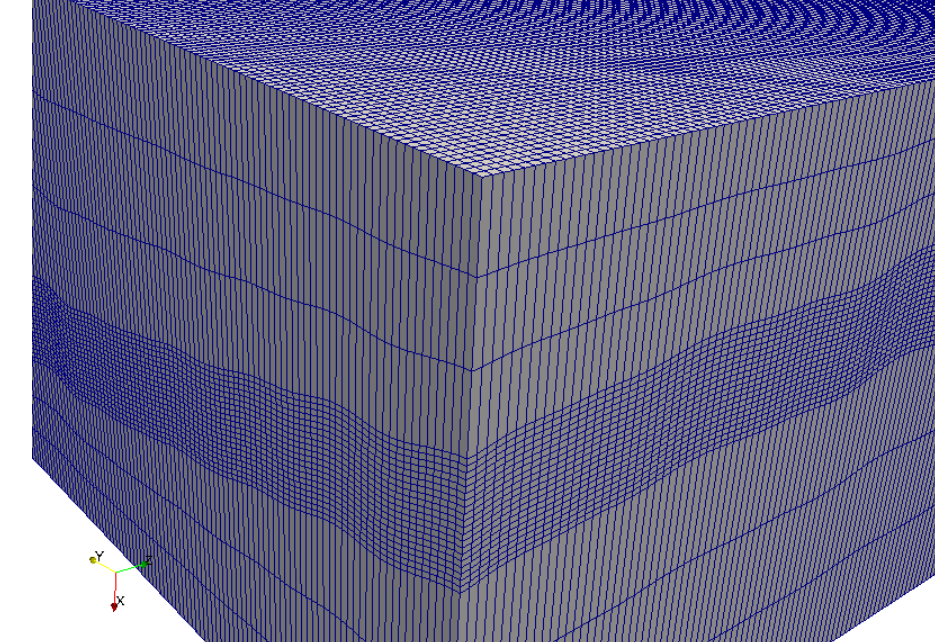
	1	2	3	4	5
Brick Geometry					
Hex Geometry					
Compositional Flow					
Linear Elasticity					
Drucker-Prager Plasticity					
Stress-dependent Permeability					

- Domain size: 150 × 1000 × 1000 ft³
- Hexahedral grid resolution: 26 × 188 × 176 = 860,288 elements
- Simulation time: 595 days, maximum $\Delta t = 1$ day

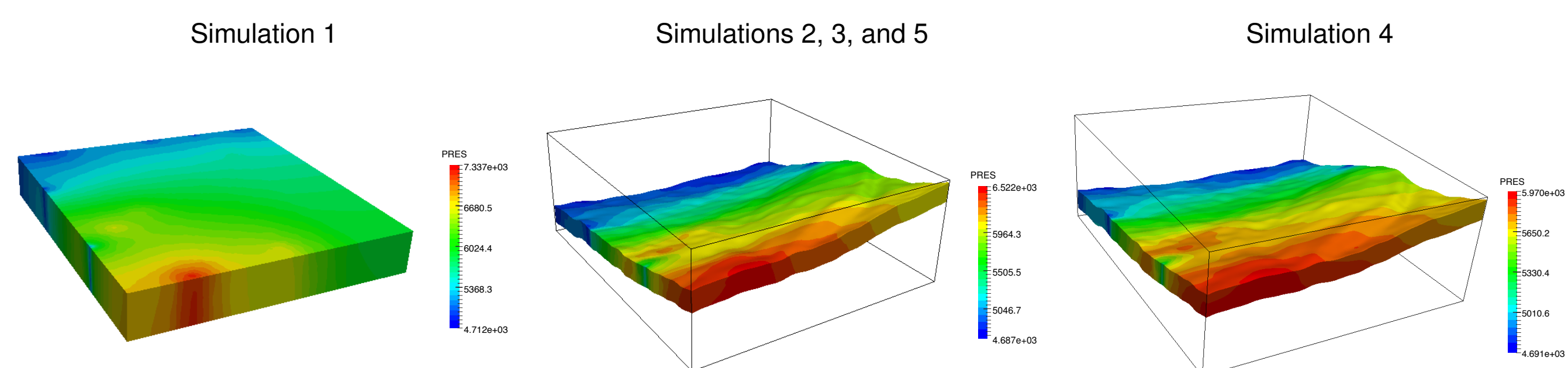
- Cranfield Domain with CO₂ Injection Formation Shaded



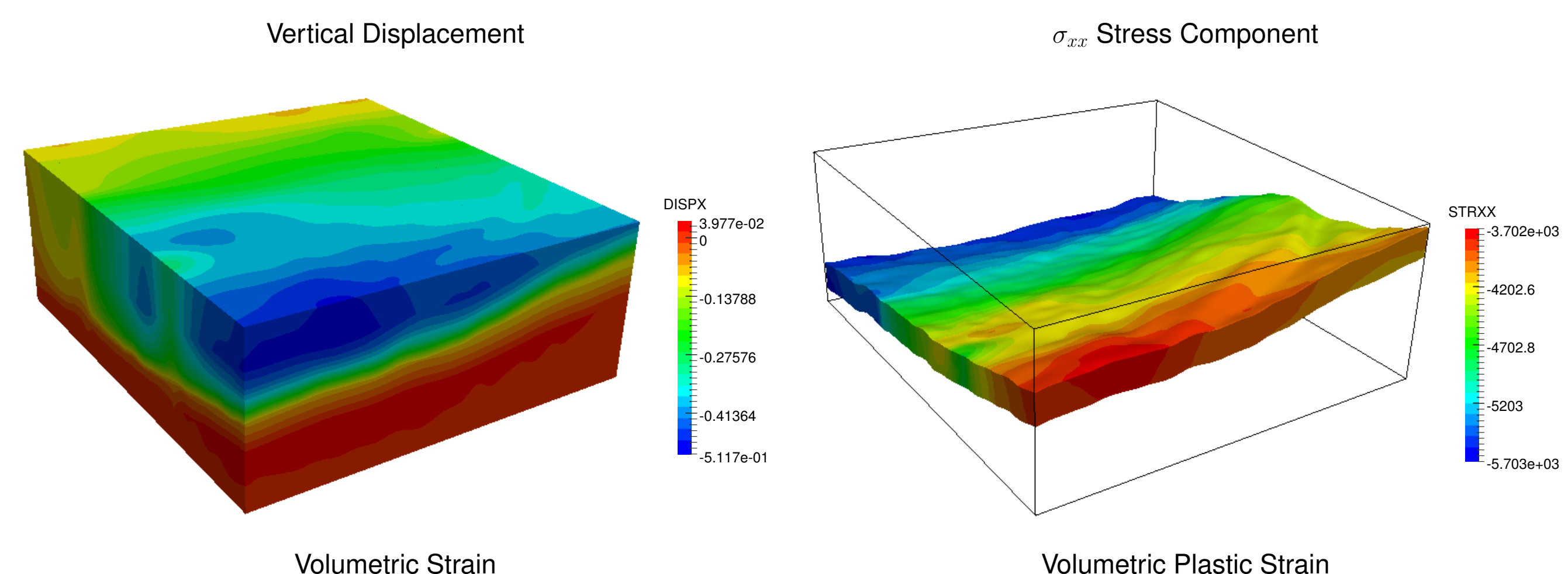
- Closeup of Hexahedral Grid Resolution



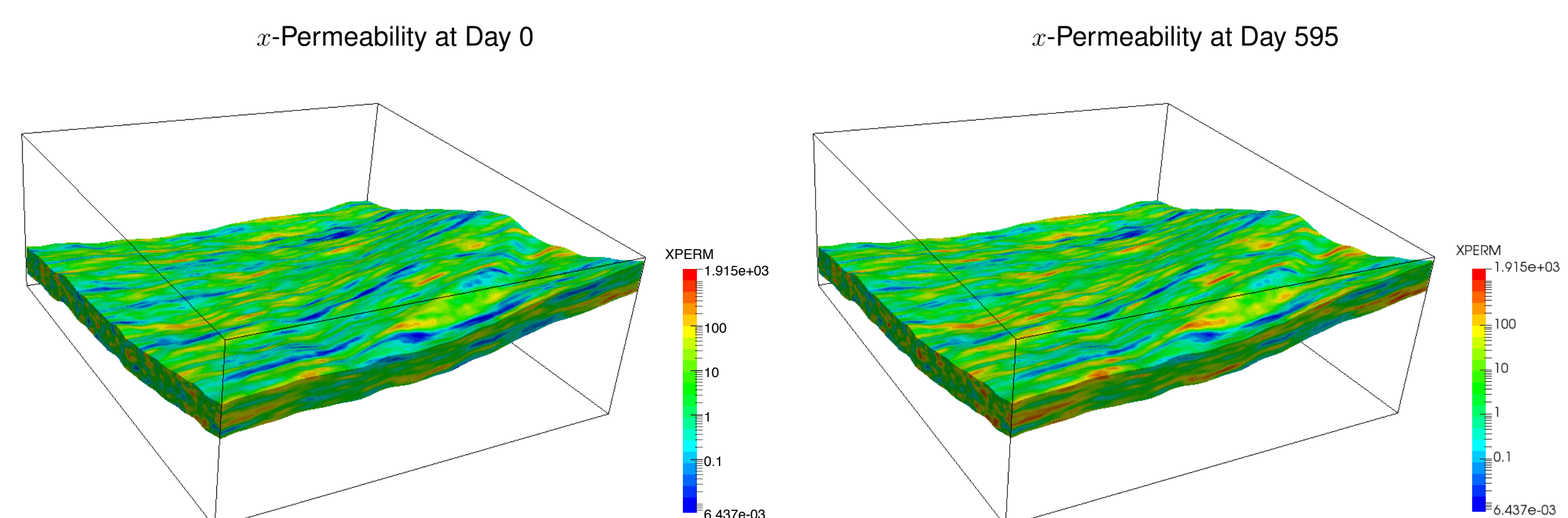
3.2 Model Comparison of Fluid Pressure at Day 595



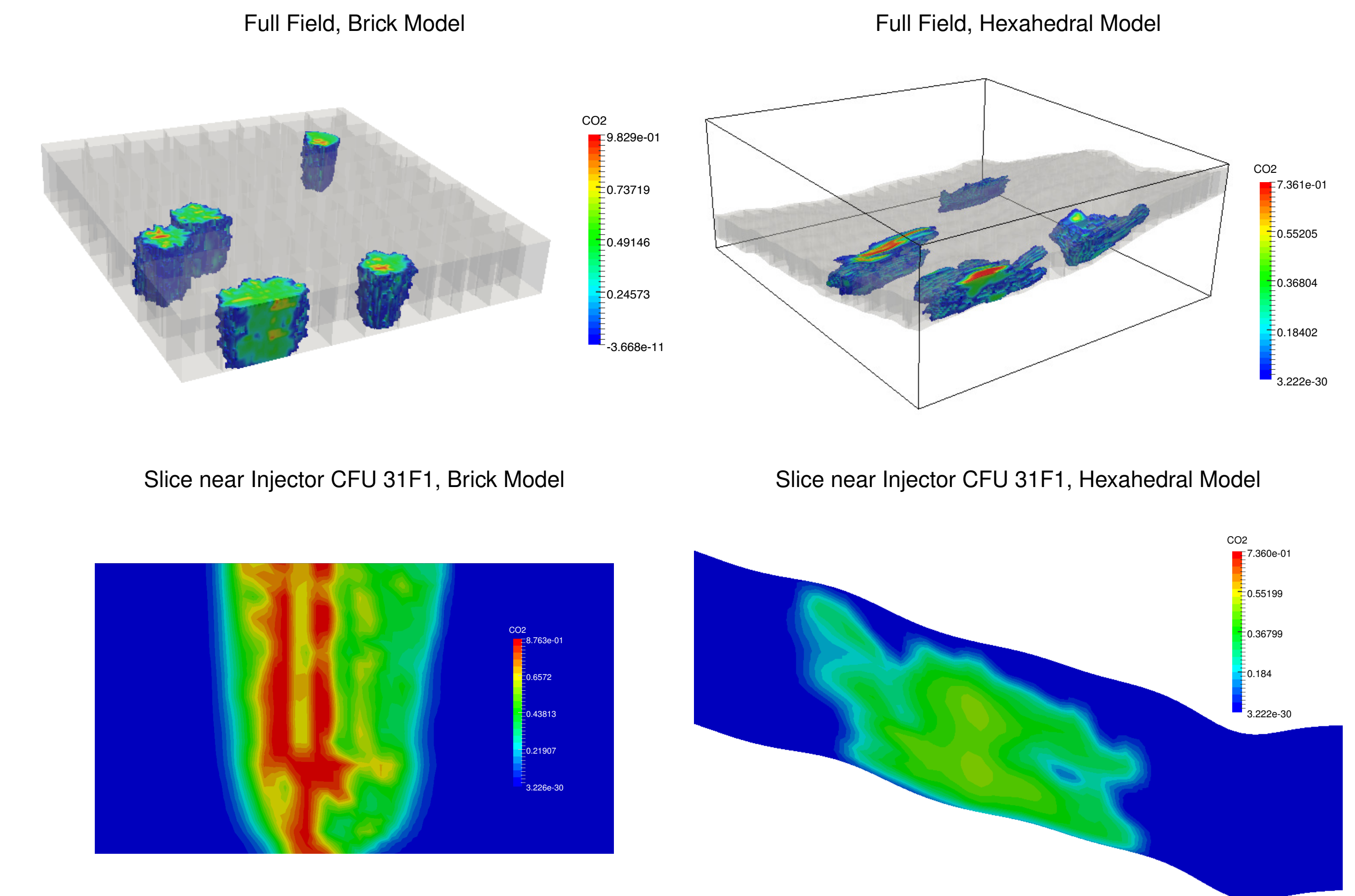
3.3 Field Results with Plasticity Model at Day 595



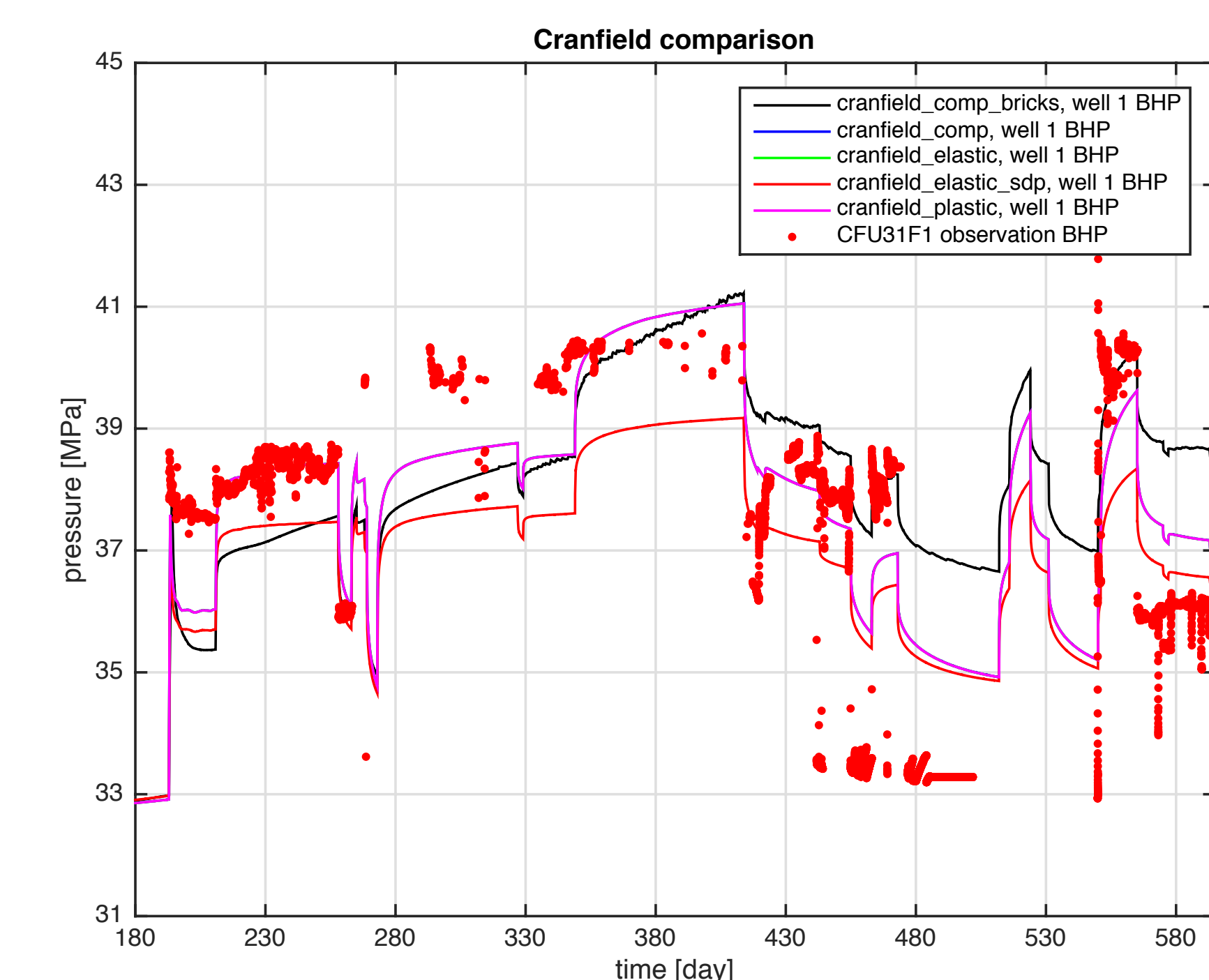
3.4 Change in Stress Dependent Permeability



3.5 Model Comparison of CO₂ Plume Migration at Day 595



3.6 Model Comparison of Bottom Hole Pressure (BHP) at Injector CFU 31F1



- Note: The curves for simulations 2, 3, and 5 appear over top of each other (blue, green, and magenta curves).

4. Conclusions and Future Work

Conclusions:

- Distorted hexahedral geometry and gravitational effects had positive impact on results.
- Mechanics allows computation of displacements and stresses. Nonlinear mechanics allows computation of plastic strain.
- Both types of mechanics did not significantly impact well BHP.
- Mechanics with stress-dependent permeability had a noticeable effect on well BHP, but model calibration is needed.

Future Work:

- Use this forward model in history matching and optimization studies.
- Use additional stress-dependent permeability models and calibrate their coefficients.
- Incorporate more accurate well information and employ local grid refinement and local time stepping techniques.
- Perform near-wellbore studies with discretely meshed well for better shear stresses and plastic effects.

References

- M. Delshad, X. Kong, R. Tavakoli, S. Hosseini, M.F. Wheeler. Modeling and simulation of carbon sequestration at Cranfield incorporating new physical models. *International Journal of Greenhouse Gas Control*, 18:463–473, 2013.
- A. Rinehart, S. Broome, P. Newell, T. Dewers. Mechanical variability and constitutive behavior of the Lower Tuscaloosa Formation supporting the SECARB Phase III CO₂ Injection Program at Cranfield Site. *Sandia Report*, 2014.
- D. White, B. Ganis, R. Liu, M. Jammoul, M.F. Wheeler, T. Dewers, et al. Control of Process for Subsurface Science and Engineering: Examples from Geologic Carbon Storage. Submitted, 2016.